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A QCD TREATMENT OF THE WEAK DECAYS OF HEAVY FLAVOUR HADRONS – WITHOUT VOODOO AND UNDUE INCANTATIONS[1]

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Abstract

There now exist several theoretical technologies for treating weak decays of heavy flavour hadrons that are genuinely based on QCD without having to invoke a deus ex machina. I focus on one of those, which employs an expansion in inverse powers of the heavy quark mass. It has developed into a rather mature framework incorporating many subtle aspects of quantum field theory. I describe its methodology for treating fully integrated decay rates as well as differential distributions, in particular energy spectra; the importance of a new type of sum rules, the SV sum rules, is emphasized. First practical benefits from this theoretical technology are listed, like predictions on lifetime ratios and extracting the KM parameter $|V(cb)|$ from inclusive and from exclusive semileptonic B decays. An outlook is given onto future developments concerning the determination of the properly defined mass of the heavy quark and its kinetic energy and a reliable extraction of $|V(ub)/V(cb)|$. A few comments on charm decays are added.

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1 Introduction

Early attempts to establish mastery over a complex problem often contain elements of Voodoo. Yet we have reached a stage where we can treat the weak decays of heavy flavour *hadrons* without such a time- honoured tool; our description can be based directly on QCD, with only one or two permissible incantations, as indicated later on. It is a three-fold message I want to give here: *significant progress* is being made which is of considerable *intellectual interest* and at the same time of increasing *practical value*, most crucially for extracting KM parameters from data. This in turn will allow us to make Standard Model (SM) predictions more precise both parametrically and numerically and thus put searches for New Physics (NP) onto a firmer basis.

Discovering CP asymmetries is seen as the ultimate prize in beauty physics. Those asymmetries being *linear* in the ratio of (coherent) amplitudes possess a high sensitivity to NP. Since the large samples of beauty hadrons expected to be accumulated at the e^+e^- B factories and ultimately at the LHC would allow measurements with a statistical uncertainty of a few percent only, the question arises whether we can acquire sufficient calculational control s.t. the basic quantities $|V(cb)|$ and $|V(ub)|$ can be determined with at most a few percent and $|V(td)|$ with 10-15 % uncertainty. Such objectives will not be attainable in the very near future or by theoretical advances alone. I anticipate it to be a rather long-term project that involves an iterative feedback between theoretical analyses and a *broad* data base as its central elements. It seems impossible to predict on which particular route this project will proceed; yet one can describe a few promising avenues, identify several gateways and list strategic elements to guide us in the future. To describe those and illustrate them through possible itineraries is the goal of my talk: in Sect.2 I sketch the Heavy Quark Expansion for fully integrated transition rates and describe some applications; in Sect.3 I extend these methods to make them applicable to differential distributions like energy spectra, and in Sect.4 I will describe a new type of powerful sum rules; in Sect.5 I discuss procedures for extracting $|V(ub)/V(cb)|$ before presenting an outlook in Sect.6.

2 $1/m_Q$ Expansions for Total Decay Widths

2.1 Methodology

The weak decay of the heavy quark Q inside the heavy flavour hadron H_Q proceeds within a cloud of light degrees of freedom (quarks, antiquarks and gluons) with which Q and its decay products can interact strongly. It is the challenge for theorists to treat these initial and final state hadronization effects. There are four Post-Voodoo theoretical technologies available, namely QCD Sum Rules, Lattice QCD, Heavy Quark

Effective Theory (HQET) and Heavy Quark Expansions. I will mainly focus on the last technique, but also indicate where there is an overlap between different technologies, with an opportunity for cooperation.

In analogy to the treatment of $e^+e^- \to hadrons$ one describes the transition rate into an inclusive final state f through the imaginary part of a forward scattering operator evaluated to second order in the weak interactions [1, 2, 3]:

$$\hat{T}(Q \to f \to Q) = i \int d^4x \{\mathcal{L}_W(x)\mathcal{L}_W^\dagger(0)\}_T \tag{1}$$

where $\{.\}_T$ denotes the time ordered product and $\mathcal{L}_W$ the relevant effective weak Lagrangian expressed on the parton level. If the energy released in the decay is sufficiently large one can express the *non-local* operator product in eq.(1) as an infinite sum of *local* operators of increasing dimension with coefficients containing higher and higher inverse powers of the heavy quark mass m_Q.[2] The width for $H_Q \to f$ is then obtained by taking the expectation value of $\hat{T}$ between the state H_Q; through order $1/m_Q^3$ one finds:

$$\Gamma(H_Q \to f) = \frac{G_F^2 m_Q^5}{192\pi^3}|KM|^2 \left[c_3(f)\frac{\langle H_Q|\bar{Q}Q|H_Q\rangle}{2M_{H_Q}} + \frac{c_5(f)}{m_Q^2}\frac{\langle H_Q|\bar{Q}i\sigma_{\mu\nu}G_{\mu\nu}Q|H_Q\rangle}{2M_{H_Q}} + \right.$$
$$\left. + \sum_i \frac{c_6^{(i)}(f)}{m_Q^3}\frac{\langle H_Q|(\bar{Q}\Gamma_i q)(\bar{q}\Gamma_i Q)|H_Q\rangle}{2M_{H_Q}} + \mathcal{O}(1/m_Q^4) \right], \tag{2}$$

where the dimensional coefficients $c_i(f)$ depend on the parton level characteristics of f (such as the ratios of the final-state quark masses to m_Q), KM denotes the appropriate combination of KM parameters, and $G_{\mu\nu}$ the gluonic field strength tensor. The last term implies also the summation over the four-fermion operators with different light flavours q. The factor $1/2M_{H_Q}$ reflects the relativistic normalization of the state $|H_Q\rangle$. It is through $\langle H_Q|O_i|H_Q\rangle$, the expectation values of the local operators O_i, that the dependence on the *decaying hadron* H_Q, and on non-perturbative forces in general, enters. Since these are matrix elements for on-shell hadrons H_Q, one sees that $\Gamma(H_Q \to f)$ is indeed expanded into a power series in μ_{had}/m_Q.

Four general remarks are in order at this point:

(A) At first one might think that the $1/m_Q$ scaling sketched above is vitiated by gluon radiation. Yet it persists to hold for fully inclusive transitions [4].

(B) The most important element of eq.(2) is – the one that is *missing*! Namely there is no term of order $1/m_Q$ in the total decay width whereas such a correction definitely exists for the mass formulae – $M_{H_Q} = m_Q(1 + \bar{\Lambda}/m_Q + \mathcal{O}(1/m_Q^2))$ – and likewise for

[2] It should be kept in mind, though, that it is primarily the energy release rather than m_Q that controls the expansion.

differential decay distributions, to be discussed later. It can be shown that integrated widths are free from $1/m_Q$ corrections due to a delicate cancellation between initial and final state hadronization effects as imposed by local colour symmetry. This can be understood in another more compact (though less intuitive) way as well: with the leading operator $\bar{Q}Q$ carrying dimension three only dimension four operators can generate $1/m_Q$ corrections; yet there is no independent dimension four operator [5, 1] *once the equation of motion is imposed* – unless one abandons local colour symmetry thus making the operators $\bar{Q}i\gamma \cdot \partial Q$ and $\bar{Q}i\gamma \cdot GQ$ independent of each other (G_μ denotes the gluon field)! The leading non-perturbative corrections to fully integrated decay widths are then of order $1/m_Q^2$ and their size is controlled by two dimension five operators, namely the chromomagnetic and the kinetic energy operators.

(C) Because non-perturbative corrections to total widths are of order $1/m_Q^2$ rather than $1/m_Q$, they amount to no more than 10 percent for B mesons: $(\mu_{had}/m_b)^2 \simeq (\mathcal{O}(1\,GeV)/m_b)^2 \sim \mathcal{O}(\%)$ (details will be given below). To predict the decay rates for beauty hadrons as a function of $V(cb)$ or $V(ub)$ with a theoretical uncertainty of less than a few percent – which is our goal –we therefore have to establish computational control over the non-perturbative corrections merely on the $\sim 20\%$ level. As a side remark: one concludes likewise that the decays of B_c mesons are mainly driven by the decays of charm rather than beauty quarks inside the B_c resulting in a lifetime considerably shorter than 1 psec. The situation is obviously numerically ambivalent for the decays of charmed hadrons; in the following I will concentrate on beauty decays with just a few remarks on charm decays.

(D) For a $1/m_Q$ expansion it is of course important to understand which kind of quark mass is to be employed there, in particular since for confined quarks there exists no a priori natural choice. It had been claimed that the pole mass can and therefore should conveniently be used. Yet such claims turn out to be fallacious [6, 7]: QCD, like QED, is not Borel summable; in the high order terms of the perturbative series there arise instabilities which are customarily referred to as (infrared) renormalons representing poles in the Borel plane; they lead to an *additive* mass renormalization generating an irreducible uncertainty of order $\bar{\Lambda}$ in the size of the pole mass: $m_Q^{pole} = m_Q^{(0)}(1 + c_1\alpha_s + c_2\alpha_s^2 + ... + c_N\alpha_s^N) + \mathcal{O}(\bar{\Lambda}) = m_Q^{(N)}(1 + \mathcal{O}(\bar{\Lambda}/m_Q^{(N)}))$. While this effect can safely be ignored in a purely perturbative treatment, it negates the inclusion of non-perturbative corrections $\sim \mathcal{O}(1/m_Q^2)$, since those are then parametrically smaller than the uncertainty in the definition of the pole mass. This problem can be taken care of through Wilson's prescriptions for the operator product expansion:

$$\Gamma(H_Q \to f) = \sum_i c_i^{(f)}(\mu)\langle H_Q|O_i|H_Q\rangle_{(\mu)} \tag{3}$$

where a momentum scale μ has been introduced to allow a consistent separation of contributions from Long Distance and Short Distance dynamics: $LD > \mu^{-1} > SD$ with the latter contained in the coefficients $c_i^{(f)}$ and the former lumped into the matrix

elements. The quantity μ obviously represents an auxiliary variable which drops out from the observable, in this case the decay width. In the limit $\mu \to 0$ infrared renormalons emerge in the coefficients; they cancel against ultraviolet renormalons in the matrix elements, yet that does *not* mean that these infrared renormalons are irrelevant and that one can conveniently set $\mu = 0$! For to incorporate both perturbative as well as non-perturbative corrections one has to steer a careful course between 'Scylla' and 'Charybdis': while one wants to pick $\mu \ll m_Q$ so as to make a heavy quark expansion applicable, one also has to choose $\mu_{had} \ll \mu$ s.t. $\alpha_S(\mu) \ll 1$; for otherwise the *perturbative* corrections become uncontrollable. Wilson's OPE allows to incorporate both perturbative and non-perturbative corrections, and *this underlies also a consistent formulation of HQET*; the scale μ provides an infrared cut-off that automatically freezes out infrared renormalons. For the asymptotic difference between the hadron and the quark mass one then has to write $\bar{\Lambda}(\mu) \equiv (M_{H_Q} - m_Q(\mu))_{m_Q \to \infty}$. This nice feature does not come for free, of course: for one has to use a 'running' mass $m_Q(\mu)$ evaluated at an intermediate scale μ which presents some technical complication. If there exists a framework other than Wilson's OPE to do the trick, I would be most eager to hear about it.

Next I want to address the question of how to determine the various expectation values. Using heavy quark expansions one can relate some of these matrix elements – and in particular those appearing in the leading terms in eq.(2) – to other observables to extract their size. We will also see that it is here where a very fruitful cooperation with analyses based on QCD Sum Rules and on Lattice QCD is emerging. Using the equation of motion one finds

$$\langle B|\bar{b}b|B\rangle/2M_B = 1 - \frac{\langle(\vec{p}_b)^2\rangle_B}{2m_b^2} + \frac{3}{8}\frac{M_{B^*}^2 - M_B^2}{m_b^2} + \mathcal{O}(1/m_b^3) \tag{4}$$

where $\langle(\vec{p}_b)^2\rangle_B \equiv \langle B|\bar{b}(i\vec{D})^2 b|B\rangle/2M_B$ describes the motion of the b quark under the influence of the gluon background field inside the B meson ($\vec{D}$ denotes the covariant derivative); here I have already invoked heavy quark symmetry to obtain:

$$\langle B|\bar{b}i\sigma \cdot Gb|B\rangle/2M_B \simeq \frac{3}{2}(M_{B^*}^2 - M_B^2) \simeq 0.74\,(GeV)^2 \tag{5}$$

For baryons Λ_b one has $\langle \Lambda_b|\bar{b}i\sigma \cdot Gb|\Lambda_b\rangle \simeq 0$. The difference $\langle(\vec{p}_b)^2\rangle_{\Lambda_b} - \langle(\vec{p}_b)^2\rangle_B$ can be deduced from the hadron masses [8]:

$$\langle(\vec{p}_b)^2\rangle_{\Lambda_b} - \langle(\vec{p}_b)^2\rangle_B \simeq \frac{2m_b m_c}{m_b - m_c}\{\langle M_B\rangle - M_{\Lambda_b} - \langle M_D\rangle + M_{\Lambda_c}\} \simeq -(0.07 \pm 0.20)\,(GeV)^2 \tag{6}$$

using present data; $\langle M_{B,D}\rangle$ denote spin averaged meson masses. The size of $\langle(\vec{p}_b)^2\rangle_B$ is not precisely known yet, beyond the inequality [9, 10]

$$\langle(\vec{p}_b)^2\rangle_B \geq \langle\mu_G^2\rangle_B \equiv \frac{1}{2}\langle B|\bar{b}i\sigma \cdot Gb|B\rangle/2M_B \simeq 0.37\,(GeV)^2 \tag{7}$$

Observable	QCD ($1/m_b$ expansion)	Data
$\tau(B^-)/\tau(B_d)$	$1 + 0.05(f_B/200\ \mathrm{MeV})^2[1 \pm \mathcal{O}(10\%)] > 1$ (mainly due to *destructive* interference)	0.98 ± 0.09
$\bar{\tau}(B_s)/\tau(B_d)$	$1 \pm \mathcal{O}(0.01)$	0.89 ± 0.20
$\tau(\Lambda_b)/\tau(B_d)$	~ 0.9	0.71 ± 0.14

Table 1: QCD Predictions for Beauty Lifetime Ratios

which is almost saturated by the QCD sum rules estimate [11] of around $0.5\,(GeV)^2$. Even present day lattice QCD should be able to extract $\langle(\vec{p}_b)^2\rangle_B$ from determining $\langle(\vec{p}_c)^2\rangle_D$ and varying the charm mass m_c [12]; likewise for $\langle B(p)|(\bar{b}_L\gamma_\mu q_L)(\bar{q}_L\gamma_\mu b_L)|B(p)\rangle$. Thus we find for the expansion parameters:

$$\frac{\langle(\vec{p}_b)^2\rangle_B}{m_b^2} \simeq 0.016 \simeq \frac{\langle\mu_G^2\rangle_B}{m_b^2}\ ,\ \frac{\langle(\vec{p}_b)^2\rangle_D}{m_c^2} \simeq 0.21 \simeq \frac{\langle\mu_G^2\rangle_D}{m_c^2} \tag{8}$$

for beauty and charm, respectively. We also have

$$m_b - m_c \simeq \langle M\rangle_B - \langle M\rangle_D + \langle(\vec{p})^2\rangle \cdot \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) \simeq 3.46 \pm 0.04\,GeV\,. \tag{9}$$

One should note that this mass *difference* is free of renormalon contributions.

2.2 Applications

With the theoretical elements thus assembled I can discuss four applications.

2.2.1 Lifetime Ratios

The predictions for the lifetime ratios of beauty hadrons are given in Table 1: There is an apparent – though not yet conclusive – discrepancy between the theoretical expectation and the present data on $\tau(\Lambda_b)/\tau(B_d)$. Therefore I want to comment briefly on the theoretical foundations for that prediction. Applying $1/m_Q$ expansions properly defined in Euclidean space to the decays of real hadrons H_Q proceeding in Minkowskian space requires certain assumptions concerning the analyticity structure of a transition amplitude; those transcend what today can rigorously be proven within QCD. Yet I consider that to constitute merely an allowed incantation since under the concept of 'quark-hadron duality' it represents an integral part of practically all applications of QCD. While it is conceivable that duality holds for semileptonic, but not for non-leptonic beauty decays, we have been unable to discern any reason for such a selective qualitative failure of duality. On the other hand, it is quite possible that the $1/m_Q$ expansion, which is actually controlled by the energy release per

Observable	QCD ($1/m_c$ expansion)	Data
$\tau(D^+)/\tau(D^0)$	~ 2	2.50 ± 0.05
$\tau(D_s)/\tau(D^0)$	$1\pm$ few %	1.13 ± 0.05
$\tau(\Lambda_c)/\tau(D^0)$	~ 0.5	0.51 ± 0.05
$\tau(\Xi_c^+)/\tau(\Lambda_c)$	~ 1.3	1.68 ± 0.5
$\tau(\Xi_c^+)/\tau(\Xi_c^0)$	~ 2.8	2.46 ± 0.75

Table 2: QCD Predictions for Charm Lifetime Ratios

quark, converges more slowly for $b \to c\bar{u}d$, $c\bar{c}s$ than for $b \to l\nu c/u$ or even $b \to u\bar{u}d$. Personally I am concerned about the short Λ_b lifetime; yet I do not consider it a theoretical desaster – unless $\tau(\Lambda_b)/\tau(B_d) \simeq 0.5$ were to hold – and I am inclined to rely on the last resort available to 'catholic' reasoning, namely to light a candle in church and pray for ultimate redemption.

The expectations [4, 13, 14] for the lifetime ratios of charm hadrons are juxtaposed to the data in Table 2: The agreement with the data is surprisingly good for a $1/m_c$ expansion; it also should be noted that quark model estimates were used for some of the *baryonic* expectation values. I will, however, present an observation below which might suggest that this agreement is somewhat coincidental.

2.2.2 Semileptonic Branching Ratio of B Mesons

The present world average yields [15]

$$BR_{SL}(B) \equiv BR(B \to l\nu X) = 0.1043 \pm 0.0024 \tag{10}$$

making it unlikely that $BR_{SL}(B)$ actually exceeds 0.11 in any significant way. A free parton model treatment leads to $BR_{SL}(b)|_{PM} \simeq 0.15$; inclusion of perturbative QCD lowers it: $BR_{SL}(b)|_{pert.QCD} \simeq 0.125 - 0.135$ [16]. The data differ from this expectation by $\sim 15 - 20\%$. Originally there was no clear need to view this difference as alarming; for a priori one would think that the non-perturbative corrections transforming $BR_{SL}(b)$ into $BR_{SL}(B)$ could naturally close the gap since they would be of order $\mu_{had}/m_b \sim 10 - 20\%$ for $\mu_{had} \sim 0.5 - 1\,GeV$. Yet we know now that the leading non-perturbative contributions arise only at the level of $(\mu_{had}/m_b)^2 \sim 1 - 4\%$. A more detailed analysis shows [1] that $BR_{SL}(B)$ is indeed lowered relative to $BR_{SL}(b)$, but only by $\sim 2\%$. There exists a loophole, though, in that analysis: the energy release in the channel $b \to c\bar{c}s$ is not large and corrections that are formally of order $1/m_b^3$ and higher might actually be numerically quite significant there. There is some theoretical evidence that they would indeed enhance $\Gamma(B \to [c\bar{c}s])$. If $\Gamma(B \to [c\bar{c}s\bar{q}]) \simeq 2 \cdot \Gamma(b \to c\bar{c}s)$ were to hold, the non-leptonic B width would be enhanced sufficiently to bring the prediction on $BR_{SL}(B)$ in line

with the data. There is one problem with such a resolution: it would raise the charm content N_c of the decay products quite significantly:

$$N_c = 1.3 \quad if \quad \Gamma(B \to c\bar{c}s\bar{q}) \simeq 2 \cdot \Gamma(b \to c\bar{c}s) \tag{11a}$$

$$N_c = 1.15 \quad if \quad \Gamma(B \to c\bar{c}s\bar{q}) \simeq \Gamma(b \to c\bar{c}s) \tag{11b}$$

Data are below, yet still compatible with $N_c = 1.15$, but not with a high value of 1.3.

To summarize the present situation [17]: there is little doubt left that experimentally $BR_{SL} \simeq 0.10 - 0.11$ indeed holds while the question of the charm content of the final state still remains somewhat unsettled; on the theoretical side it would be quite premature to invoke the lower than expected value for $BR_{SL}(B)$ as evidence for New Physics in non-leptonic B decays; on the other hand the prediction is still above the measured value although there are indications that some higher order perturbative contributions are larger than originally thought and they reduce $BR_{SL}(b)$ somewhat [18]. For proper perspective one should also keep in mind that the absolute size of the semileptonic branching ratio can be predicted with less precision than the *ratio* of semileptonic branching ratios and of lifetimes. For it – in contrast to the latter – receives both perturbative and non- perturbative contributions and a precise numerical separation of the two is not an easy task.

2.2.3 $B \to \gamma + X_{s,d}$ Transitions

Through order $1/m_b^2$ one finds [26]

$$\Gamma(B \to \gamma X_q) = \Gamma(b \to \gamma q) \left(\frac{\langle B|\bar{b}b|B\rangle}{2M_B} + f\left(\frac{m_q^2}{m_b^2}\right) \frac{\langle \mu_G^2 \rangle_B}{m_b^2} + \mathcal{O}(1/m_b^3) \right) \tag{12}$$

for $q = s, d$ with $f(m_q^2/m_b^2)$ representing a phase space factor and therefore:

$$\frac{\Gamma(B \to \gamma X_d)}{\Gamma(B \to \gamma X_s)} \simeq \frac{|V(td)|^2}{|V(ts)|^2} + \mathcal{O}(1/m_b^3) \tag{13}$$

This result is cute, yet at the same time quite useless. For it is due to the fact that the difference between m_s and m_d can be ignored on the scale of m_b; yet by the same token there is no effective kinematical distinction between X_s and X_d final states; on the other hand fully reconstructing them is not a realistic proposition.

At (formal) order $1/m_b^3$ also a serious *theoretical* problem emerges: there is a *non-local* dimension eight operator [3] generating contributions dominated by long

[3] The underlying process can be described by a diagram where a W is exchanged between the b quark and the $\bar{q}$ antiquark after a photon is emitted from the $\bar{q}$ line. There are contributions that cancel against these terms [4], yet they represent electromagnetic corrections to non-leptonic B decays and have to be counted there.

distance dynamics, in particular to $B \to \gamma X_d$; those depend on $|V(ub)|$ rather than on $|V(td)|$ (or $|V(ts)|$) and at present cannot be evaluated in a reliable fashion. A fortiori one has to be concerned that long distance dynamics will affect in particular also the exclusive channels $B \to \gamma\rho$, $\gamma\omega$ [19]. Before measurements of $BR(B \to \gamma\rho, \gamma\omega)/BR(B \to \gamma K^*)$ can be used to determine $|V(td)/V(ts)|$, one has to remove theoretically these non-Penguin contributions. The only reliable way I know of for achieving that is to measure $BR(B \to \gamma D^*)$ and/or $BR(D \to \gamma K^*)$ which do not receive any Penguin contributions; these amplitudes can then be used to gauge the size of the corresponding quantities in $B \to \gamma\rho/\omega$ and subtract them there. The left-over part represents the Penguin contribution.

2.2.4 Extracting $|V(cb)|$ from $\Gamma_{SL}(B)$ (and m_c from $\Gamma_{SL}(D)$)

The semileptonic width of B mesons can be expressed as follows:

$$\Gamma_{SL}(B) = \frac{G_F^2 m_b^5}{192\pi^3}|V(cb)|^2 \cdot$$

$$\left([z_0(x) - \frac{2\alpha_S}{3\pi}(\pi^2 - \frac{25}{4})z_1(x)][1 - \frac{\langle(\vec{p}_b)^2\rangle_B - \langle\mu_G^2\rangle_B}{2m_b^2}] - z_2(x)\frac{\langle\mu_G^2\rangle_B}{m_b^2} + \mathcal{O}(\alpha_S^2, \frac{\alpha_S}{m_b^2}, \frac{1}{m_b^3}) \right) \tag{14}$$

where the z_0, z_1 and z_2 represent known phasespace functions of $x = m_c^2/m_b^2$. To appreciate this formula one should note the following: the non-perturbative corrections are small – the leading ones arise only at order $1/m_b^2$ – and rather well known numerically: for $\langle\mu_G^2\rangle_B$ is given by the meson hyperfine splitting, see eq.(5), and there exist decent bounds on $\langle(\vec{p}_b)^2\rangle_B$. The main numerical uncertainty actually derives from the proper choice of a value for the quark mass m_b. Since m_b is raised to the fifth power it would appear that this problem introduces a large theoretical uncertainty into any attempt to extract $|V(cb)|$ from the measured semileptonic width. Yet heavy quark symmetry comes to the rescue here. It turns out that $\Gamma_{SL}(B)$ depends mainly on the difference $m_b - m_c$ rather than on m_b and m_c separately; $m_b - m_c$ is tightly constrained by the measured meson masses, see eq.(9). An independant cross check is provided by the observation that also the *shape* of the lepton spectrum is mainly controlled by $m_b - m_c$; its value can then be extracted from the data and is in full agreement with the value from eq.(8) [20] [4]. Using that information we then find [21]

$$|V(cb)|_{incl} \simeq (0.0410 \pm 0.002) \cdot \sqrt{\frac{1.5\,psec}{\tau_B}} \cdot \sqrt{\frac{BR_{SL}(B)}{0.1043}} \tag{15}$$

[4] These two methods for extracting $m_b - m_c$ are quite independant of each other. The fact that they yield a practically identical value shows that higher order corrections that have been ignored here do not make anomalously large contributions.

where the stated error is theoretical and reflects the remaining uncertainty in the size of m_b and m_c. Contrary to some recent claims in the literature, the perturbative corrections are under control when evaluated at the appropriate scale [22].

Equating the observed width $\Gamma_{SL}(D)$ with its theoretical expression (and assuming $|V(cs)| \simeq 1$) leads to the requirement "m_c" $\simeq 1.6$ GeV [23, 14]. However this is a high value relative to what is derived from charmonium spectroscopy, namely $m_c \leq 1.4$ GeV. A difference of 0.2 GeV in m_c might appear quite innocuous – till one realizes that the corresponding semileptonic width depending on m_c^5 differs by a factor of two or more! Quite generally, one might suspect higher-order perturbative and non-perturbative corrections to be large. The analysis of ref.([23]) finds however that they show a strong tendency to further *decrease* $\Gamma_{SL}(D)$ and their inclusion thus does not help at all to reproduce the measured value of $\Gamma_{SL}(D)$ with $m_c \simeq 1.4$ GeV. At present two possible conclusions can be drawn from this observation : $1/m_c$ expansions do not provide a reliable guide for charm decays since quark-hadron duality is vitiated due to 'not-so-distant cuts' in charm decays [23]; or they can be trusted – and even then only with quite a grain of salt – only for lifetime *ratios* or *ratios* of semileptonic branching ratios.

3 Energy Spectra

At first it would seem that it is beyond the reach of $1/m_Q$ expansions to describe energy spectra in beauty decays. The argument goes as follows. Consider for simplicity the photon spectrum in $B \to \gamma X$. To leading order in $1/m_b$ this decay is described by the quark level transition $b \to \gamma s$ where the γ spectrum consists of a single line located at $E_\gamma \simeq m_b/2$; gluon bremsstrahlung off the s quark generates a perturbative tail for $E_\gamma < m_b/2$. Actually for E_γ close to $m_b/2$ this gluon radiation does not represent a perturbative phenomenon. In any case one encounters the following problem: no events can be generated beyond the quark level kinematical boundary, i.e. with $E_\gamma > m_b/2$. On the other hand the true kinematical boundary is set by the higher hadron mass $M_B/2$. It would then appear that in such a treatment no events could be generated with a photon energy E_γ in the *window* $[m_b/2, M_B/2]$ – in clear conflict with observation. It is intuitively clear how this conflict gets resolved physically: the b quark is not at rest inside the B meson; its 'Fermi' motion spreads the photon line out over a region $\sim \bar{\Lambda} \equiv M_B - m_b$ thus populating also the window $[m_b/2, M_B/2]$. This intuitive parton level picture has first been introduced by the authors of ref. [24]; in ref.[25] it has been refined by carefully imposing energy-momentum conservation; I will refer to it as the AC^2M^2 model. The important new element [26, 27] (see also ref.[28]) is that this intuitive physical picture can also be realized within QCD in a rigorous fashion through a $1/m_Q$ expansion, albeit with a certain subtle, yet relevant qualification: with E_γ in the window region the mass of

the produced hadronic system is typically of order $\bar{\Lambda} m_b$, i.e. (moderately) large; yet for $m_b/2 - \bar{\Lambda}^2/2m_b \leq E_\gamma \leq m_b/2$, i.e. that highest slice of the endpoint region, this mass is of order $\bar{\Lambda}^2$ only, and one cannot trust the result of the OPE, evaluated to a finite order. It is also clear that for $m_s = 0$ the $1/m_b$ expansion is even singular around the endpoint $m_b/2$; for only in that way can non-vanishing contributions beyond the quark level kinematical boundary emerge in a heavy quark expansion. The expansion parameter is actually $\mu_{had}/[(1-y)m_b]$ with $y = 2E_\gamma/m_b$. For y not too close to unity one can compute the spectrum directly. Through order $1/m_b^2$ the non-perturbative corrections to the spectrum are expressed in terms of $\langle(\vec{p}_b)^2\rangle_B$ and $\langle\mu_G^2\rangle_B$. Applying it to semileptonic B decays one obtains a result that is very similar to a *fit* of the AC^2M^2 model to the data. This is quite remarkable since in principle there are no free parameters, although in practise there is some 'wiggle space' in the numerical size of m_b and $\langle(\vec{p}_b)^2\rangle_B$. Furthermore the region $y \simeq 1$ represents a 'black box' rather than 'terra incognita', since one knows the spectrum integrated over the endpoint region: with the total semileptonic width obtained from a non-singular expansion in $1/m_b$ and the lepton spectrum calculable for y not too close to unity one can conclude that the spectrum integrated down from unity to such values of y can be determined as well.

This can be expressed in a rather transparent way for $B \to \gamma X$. The finite spread of the photon spectrum is given by a series of $\delta^{(n)}(1-y)$, $n = 0, 1, 2, ...$, i.e. δ functions and their derivatives as a singular expansion around the quark level endpoint $E_\gamma = m_b/2$. The coefficient of the $\delta^{(n)}(1-y)$ term then represents the $(n+1)$th moment of the spectrum; these moments can then be calculated as expectation values of local operators. I will return to this point when discussing the new sum rules.

It has been shown that in QCD proper one can indeed define and calculate a function F that describes the motion of the b quark inside beauty hadrons [29, 30]. Yet this function possesses some subtle features: (i) While the motion it describes is non-relativistic, it *cannot* be expressed through a non-relativistic hadronic wavefunction; for the third and higher moments of F depend on the expectation values of *time* components of various operators. (ii) The nature of F and the way it is computed depend quite sensitively on the final state quark mass m_q (in $Q \to q$). For light quark masses – $\bar{\Lambda}^2 \sim m_q^2 \ll m_b^2$ – one obtains $F(x)$ from a light cone correlator with $x = 2(E_\gamma - m_b/2)/\bar{\Lambda}$. For heavy quarks with $m_q^2 < m_b^2$ on the other hand one expresses $F(x)$ as a temporal correlator with $x = (E_\gamma - E_0)/\bar{\Lambda}$, $E_0 = m_b(1 - m_q^2/m_b^2)/2$. That also means that the function $F(x)$ extracted from the heavy quark case *cannot* be used literally in describing the light quark case. I will return to this point below. (iii) The situation is further complicated by the fact that a third relevant scenario exists, namely for $m_q^2 \sim \bar{\Lambda} m_b$; this is the case for charm quarks! In some ways charm quarks in the final state can then superficially be described like light rather than heavy quarks; however, as I will discuss below, there exists strong circumstantial evidence that charm quarks behave more like heavy quarks in beauty decays.

It has been shown [31] that a *judiciously redefined* AC^2M^2 model implements QCD for $b \to u$ (as in $b \to l\nu u$) and $b \to s$ (as in $b \to s\gamma$) transitions in a very satisfactory way. It does not do quite as well for $b \to l\nu c$ decays as discussed later.

In summary:

• The $1/m_Q$ expansion allows to compute energy spectra in inclusive semileptonic and radiative beauty decays in terms of expectation values of local operators. It provides a very satisfactory description of the presently available data.

• Close to the kinematical endpoint one has to evaluate an increasing number of such expectation values; thus there exist at present practical limitations for computing the endpoint spectrum directly.

• Even a perfect fit to the lepton spectrum in $B \to l\nu X_c$ does not allow us *per se* to compute the lepton spectrum for $B \to l\nu X_u$ as a function of $|V(ub)/V(cb)|$ *alone*.

4 The SV Sum Rules

Semileptonic or radiative decays of heavy flavour hadrons H_Q can be viewed as the inelastic scattering of virtual W bosons or photons off an H_Q target. This general analogy had been recognized already in the early days of heavy quark symmetry [5, 32]; invoking the concept of quark-hadron duality in various forms several sum rules had been written down equating moments of observable energy spectra in inclusive decays with the corresponding quantities evaluated on the parton level [32, 33, 34, 35]. Very recently it has been shown [10] that these sum rules and a host of new ones can consistently be derived from QCD proper in a certain limit to be explained below, and that this can be achieved by pursuing the correspondence with deep-inelastic lepton-nucleon scattering: the differential decay rate is first written down in terms of (five) Lorentz-invariant functions; a $1/m_Q$ expansion allows to express those functions through the H_Q expectation values of local operators of higher and higher dimension [36]. Forming moments of these universal functions by integrating judiciously over the energy projects out certain expectation values. In deep-elastic lepton-nucleon scattering this procedure allowed to compute the evolution of such moments as a function of the momentum transfer. In heavy flavour decays one can harness heavy flavour symmetry to evaluate some of these matrix elements; this holds in particular if one can use the velocity of the hadronic system present in the decay as a second expansion parameter, i.e. in the Small Velocity (SV) limit.

We thus find the following: the sum rules mentioned above which seemed to be unrelated to each other actually represent just different moments of the same observable spectral distributions! Furthermore this approach allows to derive new sum rules as well as corrections to the previously found ones in a systematic and

comprehensive fashion. There are numerous benefits to be derived from these sum rules; I will address here the following ones: (i) They provide valuable insights into the inner workings of quark- hadron duality. (ii) They enable us to deduce the numerical value for the mass of the heavy quark and its kinetic energy from the data. (iii) They allow us to derive the inequality of eq.(7) in a field-theoretic manner. (iv) They provide us with a highly relevant bound on $|F_{B\to D^*}(0)|$, the formfactor for $B \to l\nu D^*$ at zero recoil.

ad (i):) For $B \to \gamma X_h$, where X_h denotes a heavy state making the SV limit applicable here, one finds for the deviation of the photon energy from the zeroth order line $E_0 = (m_b^2 - m_h^2)/2m_b^2$: $\langle E_\gamma - E_0\rangle \simeq \mathcal{O}(\bar\Lambda^2/m_b)$, $\langle (E_\gamma - E_0)^2\rangle \simeq \mathcal{O}(\bar\Lambda^2)$, i.e. the *center* of the photon spectrum is shifted from the original line upward by a small amount of order $\bar\Lambda^2/m_b$ with the spectrum acquiring a *spreadth* of order $\bar\Lambda$. More specifically, in the SV expansion one builds up the energy spectrum step by step: at $\mathcal{O}(v^0)$ one has only the elastic line of the free quark picture; at $\mathcal{O}(v)$ the elastic line gets shifted upward and at $\mathcal{O}(v^2)$ the height of the elastic peak is reduced with the missing contribution being re-incarnated as *inelastic* contributions due to higher resonances; perturbative gluon emission finally generates the radiative tail representing the continuum contributions from high-mass hadronic final states. The emerging picture provides us with strong circumstantial evidence that the $B \to l\nu X_c$ transition can be treated in the SV limit: for it is observed that the (quasi-)elastic channels $B \to l\nu D/D^*$ make up about two thirds of the total semileptonic width.

ad (ii): For the asymptotic mass difference $\bar\Lambda \equiv (M_B - m_b)_{m_b\to\infty}$ one derives

$$\bar\Lambda(\mu) = \frac{2}{v^2}\int_{E_0-\mu}^{E_0} dE_\gamma \frac{1}{\Gamma}\frac{d\Gamma}{dE_\gamma}(E_0 - E_\gamma) \tag{16}$$

where μ denotes the IR scale separating short and long distance dynamics as introduced through the Wilson OPE. Similarly one finds

$$\langle (\vec p_b)^2\rangle_B = \frac{3}{v^2}\int_{E_0-\mu}^{E_0} dE_\gamma \frac{1}{\Gamma}\frac{d\Gamma}{dE_\gamma}(E_0 - E_\gamma)^2 \tag{17}$$

A few comments might help to elucidate the meaning of eq.(16): Gluon emission obviously contributes to the γ spectrum in its entire domain $0 \le E_\gamma \le E_0$. Yet for E_γ very close to E_0 the emitted gluon is soft; it then is part of the nonperturbative medium where the b quark decays and thus has to be incorporated into the matrix element. This again illustrates the need for introducing the IR cut-off μ chosen to be not too small, as implied by eqs.(16,17). At the same time it is quite conceivable that the numerical dependance on the concrete value of μ is mild [6].

ad (iii): Consider the excitations in $B \to l\nu X$ produced by the vector current. For $m_l = 0$ there is no elastic channel. Positivity constraints applied at the point of

zero recoil kinematics then allows to derive – in a field-theoretical way – an inequality previously obtained through a quantum-mechanical line of reasoning:

$$\langle(\vec{p}_b)^2\rangle_B \geq \langle\mu_G^2\rangle_B \tag{18}$$

ad (iv): HQET provides us with an intriguing way to extract the KM parameter $|V(cb)|$ from exclusive semileptonic B decays. The prescription involves two steps [37, 38]: (α) One measures $B \to l\nu D^*$ decays and extrapolates to zero-recoil, thus determining $|F_{B\to D^*}(0)V(cb)|$; an average over the most recent analyses of CLEO, ALEPH and ARGUS data yield [39] :

$$|F_{B\to D^*}(0)V(cb)| = 0.0367 \pm 0.0025 \tag{19}$$

(β) For m_b, $m_c \to \infty$ one has $|F_{B\to D^*}(0)| = 1$; yet at finite quark masses one has deviations from this symmetry limit, i.e. $|F_{B\to D^*}|(0) = 1 + \mathcal{O}(\alpha_S/\pi) + \mathcal{O}(1/m_c^2) + \mathcal{O}(1/(m_b m_c)) + \mathcal{O}(1/m_b^2)$. It is important to notice that the scale of the nonperturbative corrections is set by the inverse of the *smaller* mass, i.e. the charm mass. Thus one expects corrections $\sim (\mu_{had}/m_c)^2 \simeq 0.1$ rather than the previously claimed value $\simeq 0.02$ [40] which would correspond to $(\mu_{had}/m_b)^2$. Indeed a SV sum rule yields [10]

$$1 - F^2_{B\to D^*}(0) = \frac{1}{3}\frac{\langle\mu_G^2\rangle_B}{m_c^2} + \frac{\langle(\vec{p}_b)^2\rangle_B - \langle\mu_G^2\rangle_B}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right) + \sum F^2_{B\to excitat.} \tag{20}$$

Due to eq.(18) and the positivity of the individual contributions to the inclusive rate we see that there is thus an upper bound on the formfactor for the quasi-elastic exclusive channel $B \to l\nu D^*$. [5] Including perturbative corrections omitted in eq.(20) we find $|F_{B\to D^*}(0)| < 0.94$ as a model-independant upper bound; using a value for $\langle(\vec{p}_b)^2\rangle_B$ as deduced from QCD sum rules and making a reasonable allowance for the inelastic channels we arrive at $|F_{B\to D^*}(0)| \simeq 0.90 \pm 0.03$, where our guestimate of the uncertainty reflects the fact that terms $\sim \mathcal{O}(1/m_c^3)$ have been ignored. Eq.(19) then gets translated into [21]

$$|V(cb)|_{excl} \simeq 0.0408 \pm 0.003|_{experim} \pm 0.002|_{theor} \tag{21}$$

with the first error being experimental and the second one representing the theoretical uncertainty. Clearly one has to be tremendously pleased (and relieved) that the inclusive and the exclusive analysis, eq.(15) and eq.(21), yield perfectly consistent values for $|V(cb)|$.

[5] Doubt has recently been expressed concerning the validity of this sum rule; yet that criticism is based on gross misconceptions about the derivation.

5 Extracting $|V(ub)/V(cb)|$

In Sect.3 I had already stated that at present there exist practical limitations for directly computing the lepton energy spectrum so close to the endpoint as to allow an extraction of $|V(ub)/V(cb)|$. Yet even without a future breakthrough in computational prowess there are various promising avenues to – or at least towards – that goal:

(i) To prepare the ground one extracts the parameters $\bar{\Lambda}$ and $\langle(\vec{p}_b)^2\rangle_B$ from the first and second moments of the lepton spectrum in $B \to l\nu X_c$ transitions [41], in analogy to the discussion of eqs.(16,17).

(ii) With the b quark distribution function determined through a measurement of the γ spectrum in $B \to \gamma X$ transitions, one can express the lepton spectrum in $B \to l\nu X_u$ as a function of $|V(ub)/V(cb)|$ [29, 30].

(iii) One can fit the refined AC^2M^2 model (as sketched in Sect.3) to the lepton spectrum beyond the kinematical boundary for $B \to l\nu X_c$. In ref.[31] the distribution function has been given in terms of the single parameter $\langle(\vec{p}_b)^2\rangle_B$. The fit would then also return a value for the expectation value for the kinetic energy which could then be compared with other determinations of this quantity.

(iv) However one has to face the following complication: beyond order $1/m_b^2$ there arise non-spectator contributions to $B \to l\nu X_u$ transitions; at order $1/m_b^3$ they affect charged, but not neutral B decays [42]. While their contribution to the total width is small, it is concentrated in the endpoint region. Ignoring it could then very seriously affect the value extracted for $|V(ub)/V(cb)|$. To isolate this effect one has to measure the endpoint spectrum in charged and neutral B decays separately. This is a steep price, yet presumably unavoidable if one wants to reduce the theoretical error in the value extracted for $|V(ub)/V(cb)|$ below the 10-20 % level.

6 Summary and Outlook

Our theoretical understanding of inclusive heavy flavour decays has been advanced tremendously over the last few years, and we have every reason to expect progress to continue for some time. Let me repeat just a few salient points in support of this thesis:

• Important and illuminating insights into the workings of QCD have been gained. Discussing the impact of renormalons onto heavy flavour decays nicely illustrates the level of technical sophistication that has become state-of-the-art in this field.

• New opportunities for cooperation between the various Post-Voodoo theoretical technologies have been identified and are being exploited now. It represents merely

a speed bump in the road towards progress that such opportunities for cooperation are often first seen as areas of confusion or even outright conflict.

• Significant practical gains are being made as well:

– Certain questions that only two or three years ago were beyond the scope of a theoretical discussion and could be addressed only phenomenologically can now be treated in a meaningful way, like whether the semileptonic B branching ratio is 11% or 13%; whether the lifetimes of charged and neutral B mesons differ by less than 10% or by more; what the theoretical uncertainty in the extraction of $|V(cb)|$ is, etc.

– $|V(cb)|$ has been extracted in two systematically quite different ways, namely in *inclusive* and in *exclusive* semileptonic B decays, yielding consistent values with a theoretical uncertainty that in both cases does not exceed the experimental error:

$$|V(cb)|_{incl} = 0.0410 \pm 0.002|_{experim} \pm 0.002|_{theor}$$

$$|V(cb)|_{excl} = 0.0408 \pm 0.003|_{experim} \pm 0.002|_{theor}$$

This is a major theoretical success that should be savoured appropriately [6]. Naturally we want to do even better in the future. My own feeling is that the *inclusive* method offers a higher potential for reducing the theoretical uncertainties, mainly because it requires 'only' an extrapolation of existing computational techniques coupled with a measurement of $m_b = M_B - \bar{\Lambda}$ and $\langle (\vec{p}_b)^2 \rangle_B$ as described above. For treating the *exclusive* transition with better accuracy a computational breakthrough has to be achieved to control the (non-local) contributions from the higher excitations; this is made apparent by the analysis of ref.[43]. Contrary to a recent claim [44] the perturbative corrections are under control in both the inclusive and exclusive transition [22].

– The observation that in many instances perturbative rather than nonperturbative corrections represent the main theoretical uncertainty illuminates in a nutshell the progress we have achieved. The size of perturbative corrections can be determined definitively only through a two-loop calculation; that has not been done yet. The BLM approach suggests [45] that the α_s^2 term is uncomfortably large for inclusive semileptonic B decays. Yet it is pointed out in ref.[22] that this approach is quite misleading in heavy flavour decays.

• New directions for further research have been identified whose relevance and promise had not been clear a priori. The SV sum rules can be cited as one recent example of theoretical as well as experimental interest. But there are many others:

– Analysing $B \to l\nu D^*$ for extracting $|V(cb)|$;

[6] The fact that the two central values agree so well should *not* be overinterpreted; for the stated uncertainties have at present to be considered as realistic and not overly conservative.

– Measuring the photon spectrum in radiative B decays to deduce the motion of the b quark inside B mesons;

– Observing $B \to \gamma D^*$, $l^+l^-D^{(*)}$, $D \to \gamma K^*$ to isolate non-Penguin contributions to radiative B decays;

– Studying partially integrated lepton spectra to determine the mass of the heavy quark and its kinetic energy;

– Comparing the endpoint spectra in the semileptonic decays of charged and neutral B mesons to obtain a more reliable extraction of $|V(ub)/V(cb)|$.

All of this does not mean, of course, that we have all the answers or will obtain them in a straightforward way; we will encounter unpleasant surprises. Yet it does mean that the theoretical treatment of heavy flavour decays represents no longer an embarrassment!

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